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BRIEF COMMUNICATION

TURBULENCE MODULATION IN PARTICULATE FLOWS—A THEORETICAL APPROACH

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1. INTRODUCTION

Substantial turbulence modification in particulate flows has been observed in several experimental studies. A great deal of experimental information exists in the literature on the reduction or increase of the turbulence intensity caused by the presence of a dispersed phase in a fluid (e.g. Tsuji *et al.* 1984; Lance & Bataille 1991). Gore & Crowe (1989a,b) compiled most of the available experimental data and presented the effect of the particle sizes on the reduction or enhancement of turbulence. They concluded that small particles reduced the turbulence intensity of the flow, while larger particles increased it.

On the analytical side, Owen (1969) and Hinze (1971) presented models, based essentially on order of magnitude analyses, for the interaction of turbulence with particles. Another pertinent theoretical approach, which made use of experimental data, is that of Parthasarathy & Faeth (1990) for liquid–solid flows. A review of the subject of particle–turbulence interaction (Hetsroni 1989) and two recent *ASME Symposia* (Michaelides & Stock 1989; Michaelides *et al.* 1991) expose some of the recent studies on the subject. All these studies show that the subject of particle interaction with turbulence is a very complex one, with a multitude of variables involved. It is apparent that despite the many experimental studies on dispersed multiphase flows (which includes bubbly as well as particle flows) and the progress in the numerical field, the effects of all the important variables have not yet been identified and that a solution to the problem of turbulence modification in dispersed multiphase flows is not at hand.

In general, it is known that the following six mechanisms, which are not independent of each other, contribute to the turbulence modification in dispersed two-phase systems:

(a) Dissipation of turbulent kinetic energy by the particles.
(b) Increase of the apparent viscosity due to the presence of particles.
(c) Shedding of vortices or the presence of wakes behind the particles.
(d) Fluid moving with the particle as added fluid mass to the particle.
(e) Enhancement of the velocity gradients between two rigid particles.
(f) Deformation of the dispersed phase.

Of these mechanisms, (f) is not applicable to particulate flows and the contributions of (e) and (b) are negligible in dilute particle suspensions.

This paper presents a simple mechanistic study on the turbulence modification in particle-laden flows based on the interaction of a single particle with eddies. Two predominant mechanisms for the enhancement and production of turbulence are identified: (a) the dissipation of power from an eddy for the acceleration of a particle, which appears to be the predominant mechanism for turbulence reduction; and (b) the flow velocity disturbance due to the wake of the particle or the vortices shed, which is taken as the predominant mechanism for turbulence enhancement. The effects of the two mechanisms are combined to yield the overall turbulence modification. The model, although a simplified one, exemplifies the effect of several variables, such as particle size, relative velocity, Reynolds number (Re), ratio of densities etc. A comparison with available experimental data from pipe and jet flows confirms that this simplified model predicts rather well the observed changes of turbulence intensity.

2. PARTICLE–EDDY INTERACTION

It is assumed that the turbulent flow is composed of eddies moving with a velocity u. A particle, moving with velocity v, enters an eddy and interacts with it for an amount of time, τ. The quantities u and v are, in general, vectors; however, for simplicity here they are assumed to be scalar quantities. Since the discussion will be restricted to particulate flows, where the particle-to-fluid density ratio is of the order of 1000, the effect of the history (Basset) and added mass forces in the fluid–particle interactions is neglected. Thus, the steady-state drag force, F_D, is the predominant force on the particle. The rate of work done by the fluid is equal to $F_D u$ and the change of the kinetic energy of the particle is $F_D v$. The rate of energy dissipation P is therefore $F_D(u - v)$ or

$$P = \frac{\pi c_D d^2 \rho_f (u-v)^2 |u-v|}{8}, \quad [1]$$

where c_D is the drag coefficient and d is the diameter of the particle. At finite Re_p an algebraic form for the drag coefficient, valid for a particle flowing in an unbounded medium and steady flow is $c_D = (1 + 0.15\,Re_p^{0.687})/Re_p$. This form is used in the calculations, with minor modifications due to the flow turbulence and the proximity of a particle to the walls.

During the time of particle–eddy interaction τ, when the particle finds itself in the eddy, its velocity changes because of the action of the drag force. From the equation of motion of the particle one deduces that an approximate expression for the velocity of the particle during the time interval τ is

$$v = v_0 + (u - v_0)\left[1 - \exp\left(\frac{-c_1 t}{\tau_p}\right)\right], \quad t \leqslant \tau, \quad [2]$$

where $c_1 = (1 + 0.15\,Re_p^{0.687})$; τ_p is the characteristic time of the particle, equal to $\rho_p d^2/18\mu$. Although Re_p is variable, taking c_1 as constant during the time of interaction introduces an error $< 12\%$ (Michaelides 1988) and simplifies the calculations considerably. For this reason constant c_1 is assumed in this study. The total work W during the time of interaction of the eddy with the particle τ is the integral of P with respect to time:

$$W = \frac{\pi}{12} d^3 \rho_p (u - v_0)^2 \left[1 - \exp\left(-\frac{2c_1\tau}{\tau_p}\right)\right]. \quad [3]$$

The time of interaction τ is the minimum of the eddy lifetime or the time it takes the particle to cross the eddy, i.e.

$$\tau = \min\left(\frac{l_e}{|u - v|}, \frac{l_e^2}{v}\right). \quad [3a]$$

The eddy size, l_e is determined from experimental data (Hutsinson *et al.* 1971). It was observed that in the majority (80%) of the calculations performed, the time to cross an eddy was the minimum of the two time scales.

The work W performed by the eddy on the particle is equal to the energy dissipation in the eddy. This dissipation results in a turbulence intensity reduction of the same amount. Since for the calculation of the time-average reduction in turbulence intensity we use an ensemble of particles, the term $(u - v_0)$ is approximately equal to the local relative velocity of the fluid with respect to the particle and, thus, it is replaced by U_{rel}.

Experiments show that, when $Re_p > 20$, there is an evident wake behind the particle. At $Re_p > 400$, vortices are shed behind the particles at a frequency which is a function of the Re_p (Clift *et al.* 1978). Both the wake behind the particle and the vortex shedding contribute to the velocity disturbance by the particle and are considered here as the sources of turbulence production. The change in the kinetic energy associated with the produced turbulence, therefore, is proportional to the difference of the squares of the two velocities and to the volume where the velocity disturbance occurs. Thus, the change in the kinetic energy of the fluid per unit volume is

$$\Delta E_p \sim d^2 \rho_f f(l_w)(u^2 - v^2), \quad [4]$$

where $f(l_w)$ is a function whose dimension is length and represents a measure of the region behind the particle where the fluid velocity is close to that of the particle. A quantitative estimate of this length is the effective length of the wake, l_w, or, in the case of vortex shedding, the length behind the particle where the shed vortices endure. It was assumed that the wake is half of a complete ellipsoid, with base diameter d and height l_w; the last quantity is obtained from Clift *et al.* (1978). Thus, the total amount of the kinetic energy production is

$$\Delta E_p = \frac{\pi}{12} d^2 \rho_f U_{rel}(2u - U_{rel}) f(l_w). \qquad [5]$$

The numerical term $\pi/12$ is derived from the multiplication of the volume of the ellipsoidal wake ($\pi d^2 l_w/6$) by $\frac{1}{2}$, which multiplies the kinetic energy expression. It must be noted that for very small particles $l_w = 0$ and that the production term becomes significant for larger particles.

A combination of the reduction and production term leads to the following expression for the total turbulence modification:

$$\Delta E_k = -\frac{\pi}{12} d^3 \rho_p U_{rel}^2 \left[1 - \exp\left(-\frac{2c_1\tau}{\tau_p}\right)\right] + \frac{\pi}{12} d^2 \rho_f f(l_w) U_{rel}(2u - U_{rel}). \qquad [6]$$

Equation [6] may be expanded asymptotically to determine the behavior and turbulence modification of very small or very large particles. In the case of very small particles, where $\tau \gg \tau_p$, the particle approaches the fluid velocity quickly and the wake disappears. The asymptotic expansion of [6] for fine particles is then

$$\Delta E_t = \frac{\pi d^3 \rho_p U_{rel}^2}{12}. \qquad [6a]$$

In the case of large particles, where $\tau \ll \tau_p$, the particle velocity does not change appreciably during the time of interaction and the production term predominates. At very high $Re_p (> 600)$, vortices are shed by the particles. The asymptotic expansion of the turbulence reduction term is close to zero and the turbulence modification term is equal to ΔE_p.

The asymptotic expansion shows that fine particles will cause turbulence reduction, which is proportional to the cube of the particle diameter. Large particles will predominantly cause an increase in turbulence, which is proportional to the square of the diameter. The asymptotic behavior of the gas–solid flows with particle diameter is in agreement with the compilation of data by Gore & Crowe (1989a,b). This compilation of several sets of data shows the percentage turbulence modification as a function of d/l_e. For pipe flows, where l_e is almost constant, the ordinate in the figures is proportional to d and, therefore, a direct comparison of the data with the asymptotic behavior of [6] can be made. Although the data have been obtained from several different sources and for a variety of loadings and relative velocities, it is evident that the trend of turbulence modulation with particle diameters in the figures of the two publications (including those pertinent to different eccentricity positions in pipes) is in very good agreement with the predictions of the model.

3. TOTAL TURBULENCE MODIFICATION

One may deduce from [6] the average kinetic energy modification at a point in the flow domain. For this reason, the flow at a given point is considered as an ensemble of successive eddies flowing with time-average local velocity u. Particles do not interact with each other and the probability of a particle being in an eddy depends on the local volumetric concentration of the particles. Then, from mass conservation principles, one obtains the following equation for the total time-average change in turbulent kinetic energy:

$$\Delta E_t = \frac{12\,\Delta E_k m^*}{\pi d^3 \rho_p u v} = \frac{12\,\Delta E_k m^*}{\pi d^3 \rho_p u(u - U_{rel})}, \qquad [7]$$

where m^* is the local loading (ratio of mass flow rates) of the particulate flow. It must be emphasized here that m^* may not be uniform in the flow domain, as it often happens in horizontal flows, where settling and saltation have been observed. Also, it must be pointed out that in gas–solid flows, although m^* may be high (up to 15) the volumetric concentration of particles is very low (< 1%), a fact which is in accordance with the assumptions made in this analysis.

It appears from the simplified model that there is no turbulence modulation if $U_{rel} = 0$. Although there is no clear experimental evidence on this item, intuitively it does not seem that this should be the case, because there are several variables other than U_{rel} influencing turbulence modulation. It is thought that this is due to the simplification of the mechanistic model and the approximations used, even in stipulating that $(u - v_0) = U_{rel}$.

4. COMPARISON WITH DATA

Equation [7] was compared with the experimental data of Tsuji *et al.* (1984) for a vertical pipe flow. This set of data was chosen because it is well-documented, the data were obtained at steady-state flow conditions and all variables appear explicitly in the paper or may be deduced easily. The particle diameters span the range from 0.2 to 3 mm and the loadings are < 10. Local fluid velocities were deduced from the given centerline velocities assuming a 1/7th velocity profile and (because the flow is steady-state in a vertical pipe) particle relative velocities were assumed to be equal to their terminal velocities. Because all data alluded to axisymmetric flow and no other information was given on the local concentration of the particles, uniform loading was assumed.

The drag coefficient c_D plays an important role in determining the turbulence modulation as well as the relative velocity of the particles. It is, therefore, important to use a precise expression for it. The expression presented in section 2 was used with corrections for the proximity of the particles to the wall (Clift *et al.* 1978; Happel & Brenner 1964). Given that the dimensions of the particles were much smaller than the pipe diameter ($d/D \ll 1$), it was not necessary to use a correction for the relative dimension of the particles. The drag coefficient should also be corrected for the influence

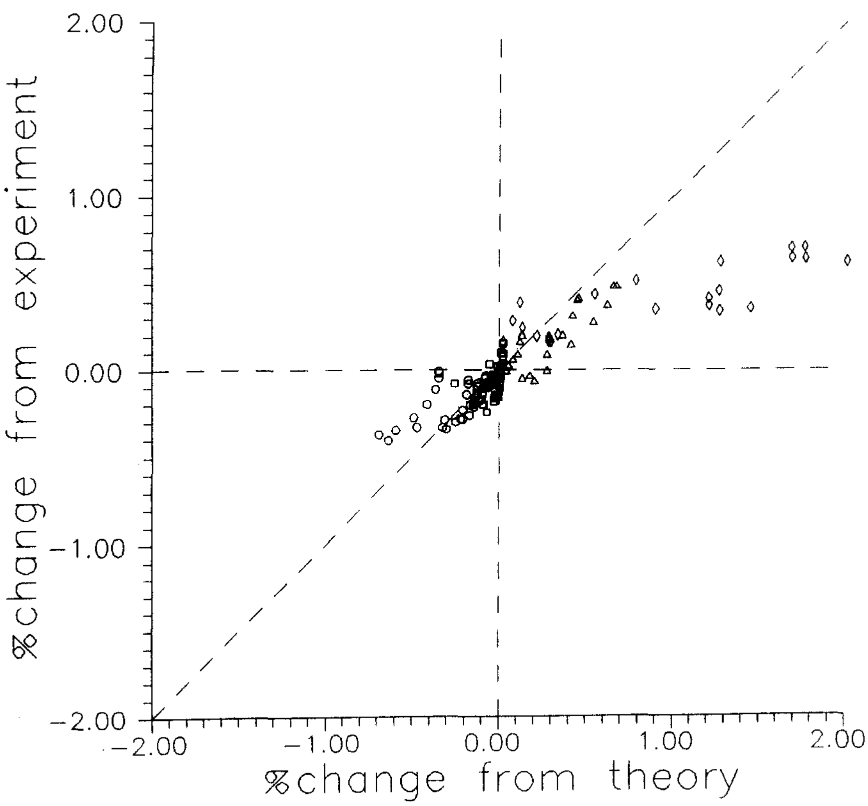

Figure 1. Comparison of theory and experimental values for pipe flow **without** correcting c_D for turbulence. Data from Tsuji *et al.* (1984): ○, $d_p = 0.2$ mm; □, $d_p = 0.5$ mm; △, $d_p = 1$ mm; ◇, $d_p = 3$ mm.

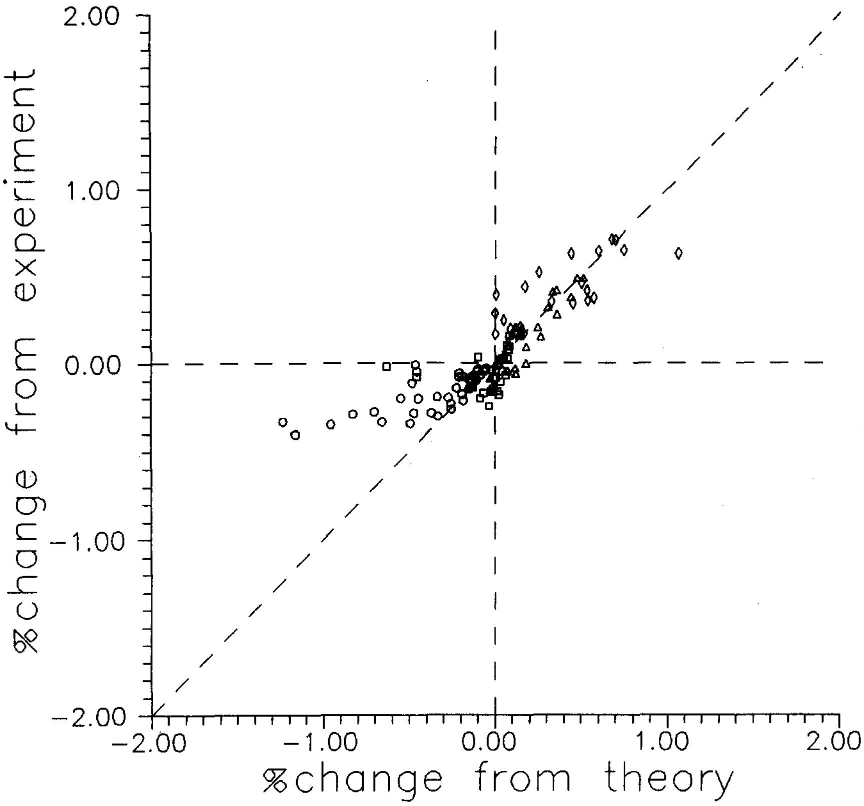

Figure 2. Comparison of theory and experimental values for pipe flow with free-flow turbulence correction for c_D. Data from Tsuji *et al.* (1984): ○, $d_p = 0.2$ mm; □, $d_p = 0.5$ mm; △, $d_p = 1$ mm; ◇, $d_p = 3$ mm.

of the local turbulence (Clamen & Gauvin 1969). Calculations and comparisons of the data were made with and without this last correction.

The figures of the experimental data by Tsuji *et al.* (1984) depict the normalized longitudinal turbulence of the gas as the particle-laden gas flow for several loadings. The figures of this paper were magnified and the turbulence intensities for the single-phase gas flow and the particle-laden flow were estimated. Thus, the experimental fractional intensity modification TM_e is calculated from the data.

The quantity $2\,\Delta E_t$ of [7], when normalized by the centerline velocity, yields the fractional turbulence modification obtained from the present model, TM_t. The theoretical and experimental turbulence modification, TM_t and TM_e were compared. Figure 1 shows a direct comparison of the fractional change in turbulence modification as predicted from [7] and as obtained from the experimental data, without taking into account the effect of free-flow turbulence on the drag coefficient. Figure 2 shows the same two quantities but with a correction on c_D (applied to the calculations for TM_t only) for the single-phase flow turbulence, obtained from Clift *et al.* (1978). The dashed line has slope equal to 1 and represents the locus of points for which $TM_e = TM_t$. Although both figures show good agreement of the results obtained from this rather simplified analysis with the experimental data, there is an obvious improvement when the single-phase flow turbulence correction is made, especially with the large particles of 3 mm. This is because at the Re_p attained by the large particles, turbulent transition in the boundary layer surrounding the particle is expected; this affects substantially the drag coefficient (Clamen & Gauvin 1969; Clift *et al.* 1978).

A comparison was also made with data obtained from vertical jets. Data from Levy & Lockwood (1981) and Modaress *et al.* (1984) were used. It was more difficult to work with this type of data because neither local loadings nor terminal velocities were reported. For comparison purposes it was assumed that the loading was uniform and that the relative velocity is equal to the terminal

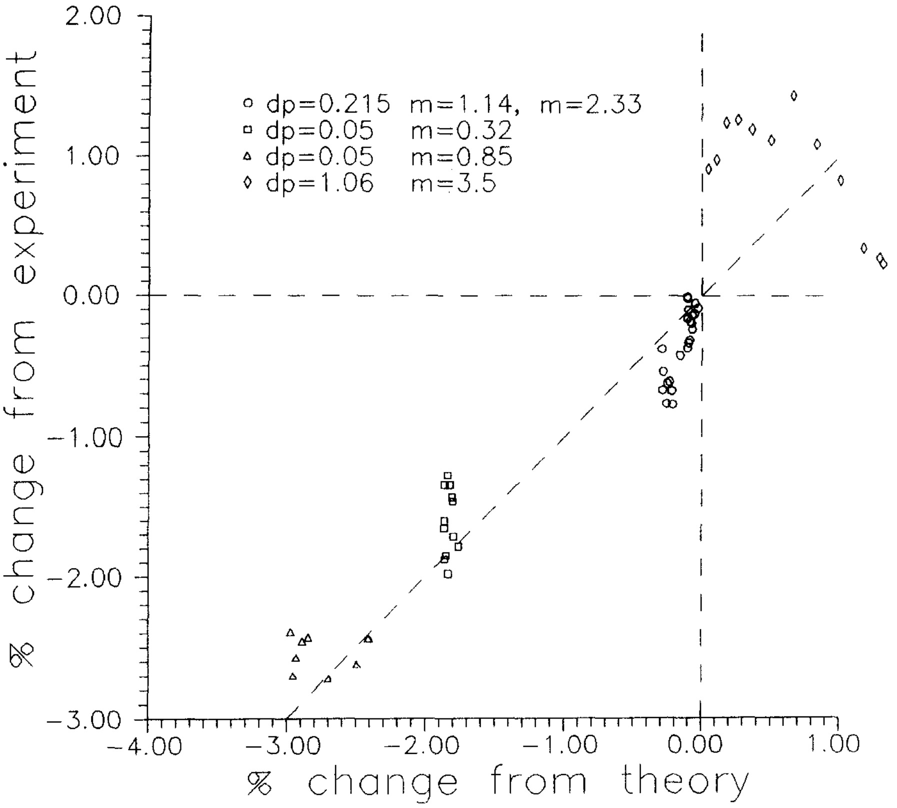

Figure 3. Comparison of theory and experimental values for jets with turbulence correction for c_D; △ and □, data from Modaress *et al.* (1984); ○ and ◇, data from Levy & Lockwood (1981).

velocity. The comparisons were made with the standard correction on the drag coefficient for the free-stream turbulence. Figure 3 depicts the results of this comparison.

Three quantitative measures of turbulence modification are the average deviation of the theoretical results from the experimental data, the average absolute deviation and the variance. Table 1 gives the values for the three quantitative measures of turbulence modification for the cases corresponding to figures 1–3.

The agreement of the theoretical results and the experimental data is very good. The average absolute error obtained is less than the experimental uncertainty of the data.

It must be pointed out that the agreement of this rather simplified theory with the experimental data is rather remarkable and perhaps a bit puzzling. It appears that the simplified theory predicts well turbulence modification in dilute particulate flows, where the steady drag force is most important in the equation of motion of the particle. Although [7] has not been tried with bubbly flows it is expected that it will not be as successful in the quantitative prediction of turbulence modification, because other forces (added mass, Basset force) and other mechanisms which are important in bubbly flows are not accounted for in the present approach. A qualitative agreement is expected, even with bubbly flow data, because the asymptotic expansion agrees with the trends observed in the compilation of all dilute dispersed multiphase flow data sets (Gore & Crowe 1989a,b).

Table 1. Percentage error in turbulence modification

	$\bar{\epsilon}$	$\overline{\lvert\epsilon\rvert}$	$\overline{\epsilon^2}$
Pipe c_D without correction for turbulence	−0.097	0.193	0.063
Pipe c_D with correction for turbulence	0.091	0.165	0.106
Jet c_D with correction for turbulence	0.021	0.399	0.169

5. CONCLUSIONS

A simplified theory is developed for the modification of turbulence intensity due to the presence of particles in dilute gas–solid flows. Two mechanisms for turbulence enhancement and reduction are examined: (1) the energy dissipation due to the acceleration of a particle is the mechanism which contributes to the reduction of the turbulence intensity of the flow; and (2) the flow disturbance due to the motion of the particle, its effective wake and the vortices shed behind, is the mechanism for turbulence enhancement. The combination of the two effects yields an expression for the total turbulence modification. A comparison of the theoretical results with available experimental data shows very good agreement. This is an indication that the two mechanisms cited above are the predominant mechanisms of turbulence modification in particulate flows and that the group of variables shown in [7] is sufficient to characterize the direction as well as the magnitude of turbulence modification.

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